Name ____ MTH 2001 - Calculus 3

PRACTICE ADVANCED STANDING EXAM

No outside resources are permitted including: notes, textbooks, cell phones or any other electronics. Show all work. Solutions without explanations will receive no points. Simplify your answers. Circle your final answers.

1. Calculate the given quantities using the following vectors:

$\vec{\mathbf{u}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{v}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

a. $2\vec{\mathbf{u}} + 3\vec{\mathbf{v}}$

b. $|\vec{\mathbf{u}}|$ and $|\vec{\mathbf{v}}|$

c. A unit vector going in the same direction as $\vec{\mathbf{u}}$

d. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$

e. $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$

f. Find the angle between the vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$

g. $\text{comp}_{\vec{\mathbf{u}}}\vec{\mathbf{v}}$

h. $\text{proj}_{\vec{\mathbf{u}}}\vec{\mathbf{v}}$

2. If $\vec{\mathbf{r}}(t) = \langle t^3, \frac{1}{\pi}\sin(\pi t), 4 + 2\ln t \rangle$, find the equation of the tangent line to the curve at the point when $t = 1$

3. Find the arclength of the curve $\vec{\mathbf{r}}(t) = \langle 3\cos t\,, 3\sin t\,, t\rangle$ between the endpoints where $t = \pi$ and $t = 4\pi$

4. [14 pts] Evaluate the following limit, or explain why it does not exist: $\lim\limits_{(x,y)\to(0,0)} \frac{10x\sin^2 y}{x^2+\sin^4 y}$

5. Use information about the gradient to answer the following about the surface

$$f(x,y) = x^2 + y^3 - 5xy$$

(a) What is the directional derivative at the point $(1,2)$ in the direction of the vector $\vec{\mathbf{v}} = \langle 3,4\rangle$

(b) What is the maximum value of the directional derivative?

(c) In what direction (given as a unit vector) is this largest directional derivative?

6. Given the function $f(x, y, z) = x^2 + yz^3 + 2xy^2$, where $x = rs\sin t$ and $y = s^2 e^t$ and $z = 3t + 2$, find the value of the partial derivatives $\frac{\partial f}{\partial r}, \frac{\partial f}{\partial s}$, and $\frac{\partial f}{\partial t}$ at the point where $r = 1$, $s = 2$, and $t = 0$.

7. Find and classify all the Critical Points of the function $f(x, y) = 3xy - x^2y - xy^2$ as Relative Maxima, Relative Minima, or Saddle Points.

8. Find the Absolute Max and Absolute Min of the function $f(x, y) = x^2 - 2xy + 2y$ on the triangular region in the *xy*-plane bounded by the points $(0,0)$ and $(2,0)$ and $(2,4)$

9. Find the Absolute Max and Absolute Min of the function $f(x,y) = x^2 + xy + y^2$ on the disk $x^2 + y^2 \leq 8$.

10. Evaluate the following integral by reversing the order of integration: $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dxdy$

11. Find the mass of a thin metal plate that occupies a region D that is bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant if the density of the plate varies according to the density function $\rho(x, y) = y$

12. Find the volume that lies inside the cylinder $x^2 + y^2 = 4$ and above the *xy*-plane and beneath the parabaloid $z = x^2 + y^2 + 1$ by using cylindrical coordinates.

13. Show that the volume of the upper half of a sphere of radius R is $\frac{2}{3}\pi R^3$ by using spherical coordinates.

14. Evaluate the following integral: $\int_0^1 \int_{4x^2}^{4x^2+1} 8x\,dy\,dx$
by making the transformations: $s = 2x$ and $t = y - 4x^2$
(Note: You *must* show the appropriate work for a change-of-variable problem. You will not receive any credit if you attempt to leave the integral in its original *xy*-form.)

15. Evaluate the following integral: $\iint_R (x - 2y)\,dA$ over the triangular region *R* that has vertices at the points $(0,0)$ and $(1,2)$ and $(2,1)$ by making the transformations: $x = 2u + v$ and $y = u + 2v$
(Note: You *must* show the appropriate work for a change-of-variable problem. You will not receive any credit if you attempt to leave the integral in its original *xy*-form.)

16. Evaluate the following line integrals by parameterizing the curves:

$\int_C 8x\,ds$ where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(2,4)$

$\int_C y^3\,dx + x^2\,dy$ where C is the arc of the parabola $x = 1 - y^2$ from $(0,-1)$ to $(0,1)$

$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}} = e^z\mathbf{i} + xz\mathbf{j} + (x+y)\mathbf{k}$ and C is given by $\vec{\mathbf{r}}(t) = t^2\mathbf{i} + t^3\mathbf{j} - t\mathbf{k}$ for $0 \le t \le 1$

17. Use the Fundamental Theorem of Line Integrals to evaluate the following integral:

$\int_C (2yz + 2x + e^y)dx + (2xz + xe^y + e^z)dy + (2xy + ye^z + \pi\cos(\pi z))dz$

where C is the line segment parameterized by the function $\vec{\mathbf{r}}(t) = 4t\mathbf{i} + (2 - 2t)\mathbf{j} + 3t\mathbf{k}$ for $0 \le t \le 1$.

18. Use Green's Theorem to evaluate the integral $\int_C 2x^2y^2dx - x^3ydy$ where C is the arc of the parabola $y = x^2$ from $(-1,1)$ to $(1,1)$ and then a line connecting $(1,1)$ to $(-1,1)$.

19. Use Stokes' Theorem to evaluate the integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}} = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$ and C is the triangle with vertices at $(1,0,0)$ and $(0,1,0)$ and $(0,0,1)$ with counterclockwise orientation when viewed from above.

20. Use the Divergence Theorem to evaluate the surface integral $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{n}} dS$ (which can also be written as $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$) where $\vec{\mathbf{F}} = (5x + 2xy)\,\mathbf{i} + (4xz - y^2)\,\mathbf{j} + (e^x - 3z)\,\mathbf{k}$ and S is the surface bounded by the parabolic cylinder $y = x^2$ and the planes $y = 0$ and $x = 1$ and $z = 3$ and and $z = 0$ with outward orientation