Name ____ MTH 1000 - Precalculus

PRACTICE ADVANCED STANDING EXAM

[#1-20 are 7 pts each & #21 is 10 pts]

1. Find the x and y-intercepts for the following:

$$x^2 = 1000 - y^3$$

x-int:

$y = 0 \Rightarrow x^2 = 1000 - 0$

$x^2 = 1000 \Rightarrow x = \pm\sqrt{1000} = \pm 10\sqrt{10}$

$\left(10\sqrt{10},0\right) \& \left(-10\sqrt{10},0\right)$

3 pts

y-int:

$x = 0 \Rightarrow 0 = 1000 - y^3$

$y^3 = 1000 \Rightarrow y = \sqrt[3]{1000} = 10$

$(0,10)$

4 pts

2. Find the equation of the line (in $y = mx + b$ form) that passes through the following points: $(2,1)$ and $(4,-5)$

$m = \frac{[-5]-[1]}{[4]-[2]} = \frac{-5-1}{4-2} = \frac{-6}{2} = -3$ 3 pts get slope

$y = mx + b \Rightarrow y = -3x + b$

$[1] = -3[2] + b \Rightarrow 1 = -6 + b \Rightarrow b = 7$ 3 pts find intercept

$y = -3x + 7$ 1 pt final equation

3. Give the domain of the following functions:

$f(x) = \frac{x-9}{x^2-x-12} = \frac{x-9}{(x+3)(x-4)}$

No zeros in denominator $\Rightarrow x \neq -3$ and $x \neq 4$ 3 pts

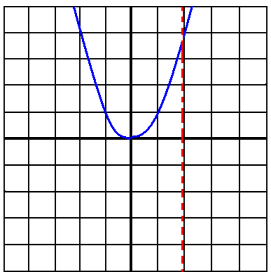
$g(x) = \sqrt{200 - 40x}$ No negatives under root $\Rightarrow 200 - 40x \geq 0 \Rightarrow -40x \geq 200 \Rightarrow x \leq -50$

(-2 pts if forget to reverse ineq)

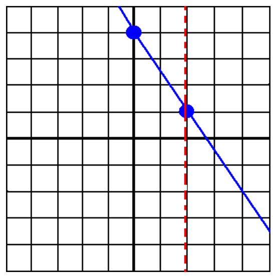
Domain : $x \leq -50$ or $(-\infty,-50]$ 4 pts final ans

4. Graph the following piecewise function: $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ -\frac{3}{2}x + 4 & \text{if } x > 2 \end{cases}$

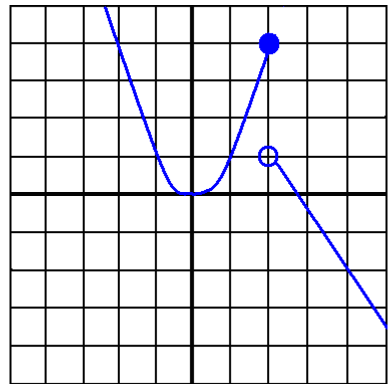
(Hint: It may help to graph the pieces separately first.)



2 pts



2 pts



3 pts

[1 pts per side & 1 pt per endpt]

5. Find the coordinates of the vertex:

$f(x) = 4(x+3)^2 + 5$

vertex : $(-3,5)$

2 pts x-term & 1 pts y-term

$f(x) = 5x^2 - 10x + 7$

$x = \frac{-[-10]}{2[5]} = \frac{10}{10} = 1$

$x = 1 \Rightarrow y = 5(1)^2 - 10(1) + 7 = 5 - 10 + 7 = 2$

vertex : $(1,2)$

2 pts x-term & 2 pts y-term

6. Divide the following polynomials and find a Quotient and a Remainder:

$(2x^3 + 7x^2 - 10x - 1) \div (2x - 1)$

2 pts first term & 1 pt each additional term

$$\begin{array}{r}
x^2 + 4x - 3 \\
2x - 1 \overline{)\, 2x^3 + 7x^2 - 10x - 1} \\
\underline{-2x^3 + 1x^2} \qquad\qquad \\
+8x^2 - 10x \qquad \\
\underline{-8x^2 + 4x} \qquad \\
-6x - 1 \\
\underline{+6x - 3} \\
-4
\end{array}$$

2 pts change all signs in subtraction

1 pt remainder

7. Identify the vertical and horizontal asymptotes:

$$f(x)=\frac{x-3}{x^2-4}=\frac{x-3}{(x-2)(x+2)}$$

$\frac{x-\ldots}{x^2-\ldots}\Rightarrow$ den deg > num deg $\Rightarrow y=0$

V.A.: $x=2$ and $x=-2$
H.A.: $y=0$

2 pts V.A. & 1 pt H.A.

$$f(x)=\frac{2x^2-3}{x^2-12x+35}=\frac{2x^2-3}{(x-7)(x-5)}$$

$\frac{2x^2-\ldots}{x^2-\ldots}\Rightarrow\frac{2}{1}=2$

V.A.: $x=7$ and $x=5$
H.A.: $y=2$

2 pts V.A. & 2 pts H.A.

8. Solve the following Inequality:

$$\frac{2}{x+2}\geq\frac{1}{x-1}$$

$$\frac{2}{x+2}-\frac{1}{x-1}\geq 0\Rightarrow\frac{2(x-1)}{(x+2)(x-1)}-\frac{1(x+2)}{(x-1)(x+2)}\geq 0$$

$$\frac{2x-2}{(x+2)(x-1)}-\frac{x+2}{(x-1)(x+2)}\geq 0\Rightarrow\frac{2x-2-x-2}{(x+2)(x-1)}\geq 0\Rightarrow\frac{x-4}{(x+2)(x-1)}\geq 0$$

$$\frac{A}{BC}\geq 0$$

3 pts move and combine into single fraction

3 pts chart/explanation

A	$-$	$-$	$-$	$+$
B	$-$	$+$	$+$	$+$
C	$-$	$-$	$+$	$+$
	-2	1	4	
	○	○	●	
	$-$	$+$	$-$	$+$

1 pt answer (either form is OK)

$-2<x<1$ and $4\leq x$
$(-2,1)\cup[4,\infty)$

9. Perform the indicated function compositions using the following formulas:

$f(x)=x+1$ $\qquad$ $g(x)=x^2-5$

$$(g\circ f)(x)=g(f(x))=g(x+1)=(x+1)^2-3=x^2+2x+1-3=x^2+2x-2$$

$$x\xrightarrow{f}x+1\xrightarrow{g}(x+1)^2-5=x^2+2x+1-5=x^2+2x-4$$ 4 pts

$$(g\circ f\circ f)(0)=g(f(f(0)))=g(f(1))=g(2)=4-5=-1$$

$$0\xrightarrow{f}(0)+1=1\xrightarrow{f}(1)+1=2\xrightarrow{g}(2)^2-5=4-5=-1$$ 3 pts

10. Find the inverse of the following function:
[Be sure to indicate if there are any restrictions on the domain of the inverse.]

$$f(x) = \sqrt{x-2}$$

$f^{-1}(x) = x^2 + 2$ 4 pts eq

Domain: $x \geq 0$ 3 pts dom

$f(x) = \sqrt{x-2} \Rightarrow$ Domain : $x \geq 2$, Range : $y \geq 0$

$$y = \sqrt{x-2}$$
$$x = \sqrt{y-2}$$
$$x^2 = y - 2$$
$$y = x^2 + 2$$

11. Solve the following equations:

$$2^{x+2} = 32$$
$$2^{x+2} = 2^5$$
$$x + 2 = 5$$
$$x = 3$$

1 pt rewrite as 2's & 1 pt proper combinations of exponents & 1 pt solve

$$\ln(x-4) = 2$$
$$\log_e(x-4) = 2$$
$$e^2 = x - 4$$
$$x = e^2 + 4$$

1 pts base *e* & 2 pts rewrite as exp & 1 pt solve

Solve for *x*:

12. $\log(x-3) + \log x = 1$

2 pts combine logs & 2 pts convert to exp & 2 pts factor poly & 1 pt ans [-1 if no discard $x = -2$]

$$\log_{10}(x-3) + \log_{10} x = 1$$
$$\log_{10}((x-3) \cdot x) = 1$$
$$\log_{10}(x^2 - 3x) = 1$$
$$10^1 = x^2 - 3x$$
$$0 = x^2 - 3x - 10$$
$$(x-5)(x+2) = 0$$
$$x = 5 \text{ or } x = -2$$

But if $x = -2$

$\Rightarrow \log_{10}(-5) + \log_{10}(-2) = 1 \Rightarrow$ Impossible

So $x = 5$ is the only solution.

13. Convert the following into the specified units:

20 degrees = ___$\frac{\pi}{9}$___ radians $\quad 20 \cdot \frac{\pi}{180} = \frac{20\pi}{180} = \frac{2\pi}{18} = \frac{\pi}{9}$ 3 pts

$\frac{\pi}{18}$ radians = __10°__degrees $\quad \frac{\pi}{18} \cdot \frac{180}{\pi} = \frac{180}{18} = 10°$ 4 pts

14. Find the exact value of the following:
[Note: The angles are in radians.]

$\sec\frac{3\pi}{4} = -\sqrt{2}$

$\frac{3\pi}{4} \Rightarrow \frac{\pi}{4}$ in QII $\Rightarrow (-,+)$

$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} \Rightarrow \sec\frac{3\pi}{4} = -\frac{\sqrt{2}}{1}$

2 pts # & 1 pt +/- sign

$\cot\frac{7\pi}{3} = +\frac{1}{\sqrt{3}}$

$\frac{7\pi}{3} \Rightarrow \frac{\pi}{3}$ in QI $\Rightarrow (+,+)$

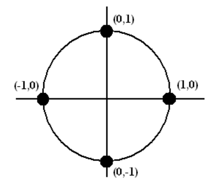
$\tan\frac{7\pi}{3} = +\sqrt{3} \Rightarrow \cot\frac{7\pi}{3} = +\frac{1}{\sqrt{3}}$

2 pts # & 1 pt +/- sign

$\sin(4\pi) = 0$

$4\pi \Rightarrow 2$ rev

1 pt

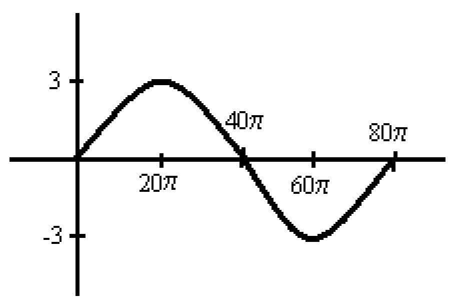


15. Graph the following trig function:
Be sure to label your axes appropriately. [Note: The angles are in radians.]

$f(x) = 3\sin\left(\frac{1}{40}x\right)$

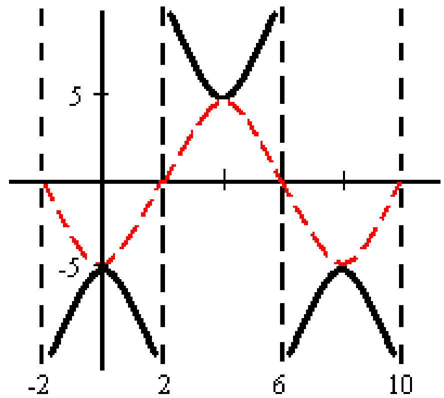
Amp: 3

Period: $\frac{2\pi}{\frac{1}{40}} = 2\pi \cdot \frac{40}{1} = 80\pi$



2 pts shape
2 pts amp
2 pts period
1 pt labeling appropriately

16.



Write an equation that describes the above graph:

[Note: The angles are in radians and there is no phase shift.]

Amp : 5

Period: $8 = \frac{2\pi}{B} \Longrightarrow 8B = 2\pi \Longrightarrow B = \frac{2\pi}{8} = \frac{\pi}{4}$

$f(x) = -5\sec\left(\frac{\pi}{4}x\right)$

2 pts amp term & 2 pts trig function & 3 pts period/angle term

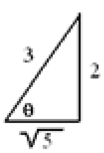
17. Find the exact value of the given trig function:
(Note: The angles are measured in radians.)

$\cos\left[\cos^{-1}\left(\frac{3}{2}\right)\right] =$ Does Not Exist 2 pts

$\frac{3}{2} > 1$ and $\cos\theta \le 1$

$\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] = \frac{2\pi}{3}$ 2 pts

$\frac{4\pi}{3}$ in QIII $\Rightarrow \cos\theta = - \Rightarrow$ move to QII



$\cos\left[\tan^{-1}\left(-\frac{2}{3}\right)\right] = -\frac{\sqrt{5}}{3}$ 3 pts

$\tan^{-1}\left(-\frac{2}{3}\right) = \theta \Rightarrow \tan\theta = -\frac{2}{3} \Rightarrow \frac{\text{OPP}}{\text{ADJ}} = \frac{2}{3}$ & QIV $(+,-)$

$\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = -\frac{\sqrt{5}}{3}$

18. Prove the following trigonometric identity:

$\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \csc\theta$

2 pts common denom & 2 pts $s^2 + c^2 = 1$ & 2 pts factor/cancel & 1 pts final ans

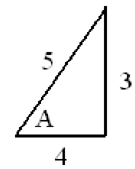
$$\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

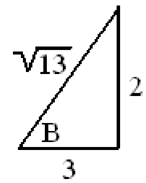
$$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)} + \frac{\sin\theta\cdot\sin\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)} + \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta+1}{\sin\theta(1+\cos\theta)} = \frac{1}{\sin\theta} = \csc\theta$$





Find the exact value of the following:

19. $\sin\left(\cos^{-1}\left[\frac{4}{5}\right]+\tan^{-1}\left[\frac{2}{3}\right]\right)=$

Use the following formulas to help answer the question above:

Angle Sum & Difference Formulas:
$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$
$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\cos A=\frac{4}{5}$ $\quad$ $\tan B=\frac{2}{3}$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$ 3pts

$$=\left[\frac{3}{5}\right]\left[\frac{3}{\sqrt{13}}\right]+\left[\frac{4}{5}\right]\left[\frac{2}{\sqrt{13}}\right]=\frac{9}{5\sqrt{13}}+\frac{8}{5\sqrt{13}}=\frac{9+8}{5\sqrt{13}}=\frac{17}{5\sqrt{13}}$$

3 pts $\quad$ 1 pt

20. Find all solutions in the interval $0\le\theta<2\pi$:
[Note: The angles are measured in radians.]

$$2\sin^2\theta+5\sin\theta-3=0$$

$$(2\sin\theta-1)(\sin\theta+3)=0$$

$$\sin\theta=+\frac{1}{2} \text{ or } \sin\theta=-3$$

$$\theta=\frac{\pi}{6} \text{ in QI \& QII or } \theta=D.N.E$$

$$\theta=\frac{\pi}{6} \text{ and } \frac{5\pi}{6}$$

2 pts factor quad & 1 pt no solution term & 2 pts each answer
[1 pt ref ang & 1 pt quad]

21. Find the value of θ [in radians] in the First Quadrant where $\cos\theta = \frac{1}{2}$, then find the values of the other five trig functions for that same angle θ.

$\text{QI} \Rightarrow (+,+) \Rightarrow \sin = + \ \& \ \cos = + \ \& \ \tan = +$

$1^2 + y^2 = 2^2$

$\cos\theta = \frac{1}{2}$ $\qquad \theta = \frac{\pi}{3}$ 1 pt

$1 + y^2 = 4$

$\cos\theta = \frac{1}{2} \Longrightarrow \theta = 60° \text{ or } \frac{\pi}{3}$

$y^2 = 3$

$y = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$ 3 pts

$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$ 3 pts

$\sec\theta = \frac{2}{1} = 2$ 1 pt

$\csc\theta = \frac{2}{\sqrt{3}}$ 1 pt

$\cot\theta = \frac{1}{\sqrt{3}}$ 1 pt